Nondiffractive fields

J. Bajer and R. Horák
Department of Optics, Palacký University, tř. Svobody 26, 771 46 Olomouc, Czech Republic
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At the present time no general definition of nondiffractive beam has been generally accepted. We propose one simple definition based on the Poynting vector and light intensity. [S1063-651X(96)13709-0]

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Diffraction of light was first reported by Leonardo da Vinci, but the first steps to its understanding were only made in the 17th (Grimaldi, Huygens, Hook, Newton) and 18th centuries (Fresnel, Young). Historically, diffraction is what we call the phenomenon when light is not traveling in straight lines although it should be according to the laws of ray optics. The discovery of diffraction served as an important argument for overcoming corpuscular Newtonian optics. The present-day idea about diffraction is not connected with obstacles and apertures only [1]. Any nonhomogeneous light intensity distribution makes for a diffraction of light (remember the Gaussian beam in free vacuum).

In a narrower ''technical'' sense, diffraction means the defocusing of a beam–i.e. the transversal change of the energy distribution in the beam. For the light energy and information transfer it is very important to suppress that defocusing or energy melting maximally. That problem was a strong motivation for the study of nondiffractive fields.

Diffraction is the natural attribute of any wave phenomenon. Our goal will be to give an appropriate definition or condition for the stationary light fields propagating with no transversal spreading of energy in any dielectric media (nonhomogeneous and nonlinear as well).

Some nondiffractive solutions are already known [2]. In a vacuum they are planar waves and Bessel beams [3–8]; in waveguides these fields are known as well, and are called modes, in nonlinear media they are spatial solitons [9], etc.

Up to now no general definition of a nondiffractive beam has been accepted. We want to discuss some possible approaches to this question, and propose one simple definition based on the Poynting vector.

The simplest intuitive definition of nondiffractive beams can be introduced as a field with no transversal energy flux, i.e., $\mathbf{I}_T=\mathbf{0}$ (the trivial definition). We will show that more general nondiffractive fields are possible, and that we need find a more general definition that includes the nondiffractive fields with nonzero transversal energy flux.

The real photodetector measures the time-averaged light power, which is given as product of the detector area and light intensity $\mathbf{I}=\langle\mathbf{S}\rangle=\langle\mathbf{E}\times\mathbf{H}\rangle$. The quantity $\mathbf{S}=\mathbf{E}\times\mathbf{H}$ is a Poynting vector. We point out one important property of the definition of the Poynting vector, that it is very general and that it holds in any kind of possible media — linear, nonlinear, nonhomogeneous, or anisotropic — while the definition of the energy density depends strongly on media properties.

We will confine our considerations to stationary light fields (i.e., monochromatic beams), and call the assumed frequency ω. The electric intensity $\mathbf{E}$ can be written as a sum of complex components

$$\mathbf{E}=\vec{\mathcal{E}}\exp(-i\omega t)+\vec{\mathcal{E}}^{*}\exp(i\omega t). \tag{1}$$

The same time behavior is assumed for the other electric and magnetic vectors $\mathbf{D}$, $\mathbf{H}$, and $\mathbf{B}$.

The light intensity then reads

$$\mathbf{I}=\langle\mathbf{S}\rangle=\langle\mathbf{E}\times\mathbf{H}\rangle=2\,\mathrm{Re}\{\vec{\mathcal{E}}^{*}\times\vec{\mathcal{H}}\}=\frac{2}{\mu\omega}\mathrm{Im}\{\vec{\mathcal{E}}^{*}\times\mathrm{rot}\vec{\mathcal{E}}\}, \tag{2}$$

and the energy conservation law reads

$$\mathrm{div}\mathbf{I}=\mathbf{0}. \tag{3}$$

The vector $\mathbf{I}$ can be decomposed as $\mathbf{I}=\mathbf{I}_T+\mathbf{I}_L$, where $\mathbf{I}_T$ is the transversal component of $\mathbf{I}$ (i.e., the component laying in the x,y plane) while the longitudinal part $\mathbf{I}_L$ is the z component; we will suppose the light propagation to be in the z axis only in the following.

In waveguide theory we can use the modal function ansatz

$$\vec{\mathcal{E}}(x,y,z)=\mathbf{e}(x,y)\exp(i\beta z) \tag{4}$$

to find the explicit form of modes propagating along the z axis, and thus can also serve as a useful definition of nondiffractive beams in the waveguide.

With respect to the chosen propagation axis, we assume the transversal nonhomogeneity of a waveguide described by the dielectric permittivity $\epsilon(x,y)$. So the electric induction $\vec{\mathcal{D}}=\epsilon\vec{\mathcal{E}}$ will have the same form (4), and the magnetic intensity as well. This can be proved by simple vector algebra $\vec{\mathcal{H}}(x,y,z)=\mathrm{rot}\vec{\mathcal{E}}/i\omega\mu=\mathrm{rot}[\mathbf{e}(x,y)\exp(i\beta z)]/i\omega\mu=\mathbf{h}(x,y)\exp(i\beta z)$ and the light intensity vector $\mathbf{I}=2\mathrm{Re}\{\mathbf{e}^{*}\times\mathbf{h}\}=\mathbf{I}(x,y)$ does not depend on z. The energy conservation law $\mathrm{div}\mathbf{I}=\mathbf{0}$ implies $\mathrm{div}\mathbf{I}_T=\mathbf{0}$, which inspired us to introduce the following definition of a nondiffractive field. *If the average transversal component of the Poynting vector $\langle\mathbf{S}_T\rangle=\mathbf{I}_T$ of an optical beam for all x, y, and z fulfills the condition*

$$\mathrm{div}\mathbf{I}_T=\mathbf{0}, \tag{5}$$

then such a beam is diffraction free.

It follows from this definition and the conservation law (3) that $\mathbf{I}_L=\mathbf{I}_L(x,y)$ for any nondiffractive beam. The transversal component $\mathbf{I}_T$ depends on z in general, but not in the case of linear homogeneous media, as can be proved using the decomposition of field to plane wave components. Using

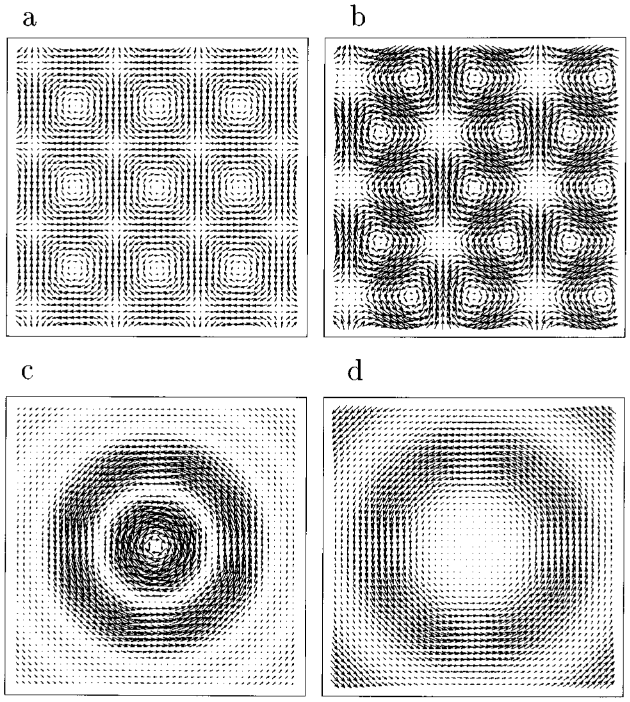

FIG. 1. The needle plot of transverse light energy flows $\mathbf{I}_T$ for (a) $\mathcal{N}=4$, (b) $\mathcal{N}=3$, and (c) $\mathcal{N}=40$, and phases $\phi_n=2\pi n/\mathcal{N}$, (d) $\mathcal{N}=40$, and $\phi_n=8\pi n/\mathcal{N}$.

the same decomposition, one can prove that for linear homogeneous media (vacuum) both definitions of nondiffractive fields (4) and (5) are equivalent.

It is worthwhile to point out that our definition also has simple geometrical interpretation: The transverse light intensity component $\mathbf{I}_T$ of a nondiffractive field has no source, or, equivalently, all power lines of this field are closed. This is clearly seen in Figs. 1(a)–1(d).

This definition includes the most general nondiffractive fields, namely, all fields with zero transversal flux, $\mathbf{I}_T=\mathbf{0}$, as well as those that fulfill the modal function ansatz and maybe some others. We confirm our claims as follows.

There exist nondiffractive beams fulfilling definition (5) which do not have the form of (4). One can simply prove that the standing wave in this case, $\vec{\mathcal{E}}=\hat{\mathbf{x}}(Ae^{i\beta z}+Ae^{-i\beta z})=\hat{\mathbf{x}}2A\cos\beta z$, $\vec{\mathcal{H}}=\hat{\mathbf{y}}2A\sin\beta z/i\omega\mu$, has $\langle\mathbf{I}\rangle=\mathbf{0}$. For completeness we add that the instantaneous Poynting vector is $\mathbf{S}=\hat{\mathbf{z}}\sqrt{\epsilon/\mu}A^2\sin2\beta z\,\sin2\omega t$. So we have proved that our definition is more general than (4).

At the end we shall give a nontrivial demonstration of nondiffractive fields. We present nondiffractive fields with transversal energy fluxes $\mathbf{I}_T\neq\mathbf{0}$, e.g., beams with a honeycomb transversal flux of energy [see Figs. 1(a) and 1(b)] or with a spiral transversal flux of energy [see Fig. 1(c) and 1(d)]. These fields are not so ''exotic'' as one might think. We will now present one such nondiffractive beam analytically. That field is constructed from four monochromatic plane waves of the same amplitude A; its wave vectors lie on cones with angle θ, the wave-vector ends form a regular square, and the polarization of every plane wave has radial direction. The phases of the waves are shifted by $\pi/2$ to its neighbor. It is very important to note that in the case of the same phases, no transversal flux exists.

The complex vectors of elementary plane waves can be found as follows:

$$\vec{\mathcal{E}}_1=\begin{pmatrix}\cos\theta\\0\\-\sin\theta\end{pmatrix}Ae^{i\kappa x}e^{i\beta z}e^{i0},$$

$$\vec{\mathcal{E}}_2=\begin{pmatrix}0\\\cos\theta\\-\sin\theta\end{pmatrix}Ae^{i\kappa y}e^{i\beta z}e^{i\pi/2},\tag{6}$$

$$\vec{\mathcal{E}}_3=\begin{pmatrix}-\cos\theta\\0\\-\sin\theta\end{pmatrix}Ae^{-i\kappa x}e^{i\beta z}e^{i\pi},$$

$$\vec{\mathcal{E}}_4=\begin{pmatrix}0\\-\cos\theta\\-\sin\theta\end{pmatrix}Ae^{-i\kappa y}e^{i\beta z}e^{i3\pi/2}.\tag{7}$$

The sum of these elementary plane waves for the resulting intensity gives

$$\vec{\mathcal{E}}=\vec{\mathcal{E}}_1+\vec{\mathcal{E}}_2+\vec{\mathcal{E}}_3+\vec{\mathcal{E}}_4=2A\begin{pmatrix}-\cos\theta\,\cos\kappa x\\i\cos\theta\,\cos\kappa y\\-\sin\theta(\sin\kappa y-i\,\sin\kappa x)\end{pmatrix}e^{i\beta z}.\tag{8}$$

The divergent angle θ is bound by the wave-vector transversal component κ, and the longitudinal one β by $\tan\theta=\kappa/\beta$. One can simply verify that the resulting field is transversal, i.e., $\mathrm{div}\vec{\mathcal{E}}=0$. The light intensity can be calculated using formula (2), and after some tedious algebra manipulations one obtains

$$\mathbf{I}=8A^2\left(\frac{\epsilon}{\mu}\right)^{1/2}\begin{pmatrix}-\sin\theta\,\cos\kappa x\,\sin\kappa y\\\sin\theta\,\sin\kappa x\,\cos\kappa y\\-\cos\theta(\cos^2\kappa x+\cos^2\kappa y)\end{pmatrix}.\tag{9}$$

We want to hint at the nonzero transversal components of light intensity vector. The electric field has the modal form (4), and also satisfies definition (5); nevertheless it has complicated transversal energy fluxes, as shown in Fig. 1(a), resembling a squared honeycomb. The equation for power lines can be derived as $|\sin\kappa x||\sin\kappa y|=K$; from this we can see space-periodic behavior.

In Figs. 1(a)–1(d), using the computer, we have displayed four examples of similar nondiffractive beams, with complex transversal energy flows expressed by small arrows (needle plot). These fields are constructed from $\mathcal{N}$ plane waves of the same frequency and amplitude

$$\mathbf{E}=\sum_{n=1}^{\mathcal{N}}A\vec{e}_n\exp[i\mathbf{k}_n\cdot\mathbf{r}-i\omega t+\phi_n],\tag{10}$$

where wave vectors lie on the surface of cones and their ends create regular planar N-angle polygons in the transverse plane xy,

$$\mathbf{k}_n=\begin{pmatrix}\kappa\cos(2\pi n/\mathcal{N})\\ \kappa\sin(2\pi n/\mathcal{N})\\ \beta\end{pmatrix}, \tag{11}$$

and the polarization was chosen as radial,

$$\vec{e}_n=\frac{\mathbf{k}_n\times(\mathbf{k}_n\times\hat{\mathbf{z}})}{|\mathbf{k}_n\times(\mathbf{k}_n\times\hat{\mathbf{z}})|}. \tag{12}$$

The phase increases monotonically with $\phi_n=2\pi n/\mathcal{N}$ in Figs. 1(a)–1(c) or $\phi_n=8\pi n/\mathcal{N}$ in Fig. 1(d). Figures 1(c) and 1(d) strongly resemble the Bessel beams J_1, although we obtained them with only 40 plane waves. Note that the phases [Figs. 1(c) and 1(d)] have a strong influence on the distribution and orientation of the light intensity flux, and that all the power lines are closed, as expected.

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